

Global Warming Potential of Green House (GH) Gas
Release at Different Altitudes

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Abstract

Many human activities involve fossil fuel combustion that releases GH gases. Emissions from ground transportation occur at earth's surface. Tall chimney's of power plants and smaller private planes release at higher altitudes. Subsonic jet aircraft release in the **UT-LS** (Upper Troposphere, Lower Stratosphere).

Short term analysis ignoring atmospheric circulations indicates that the transient effect of a single release of GH gases on global warming, grows with the altitude of release.

Because of rapid mixing in the lower troposphere caused by circulations, there are only two altitudes to compare for longer term effects, the UT-LS, and anywhere in the lower troposphere. Jet aircraft flying is the only activity releasing directly in the UT-LS. Our limited computer runs took only H_2O releases into account, and did not consider the formation of condensation trails and cirrus clouds. They indicate that the present releases in the UT-LS cause a slightly larger reduction in the amount of terrestrial radiation escaping to space, than if they were to occur in the lower troposphere. So, the northern mid-latitudes where majority of jet aircraft releases occur today is a sensitive region for such releases.

This suggests an important topic for further study. Polar ice melt at the north pole is faster than at the south pole. Whether the large volumes of jet aircraft releases in northern mid-latitudes are at least partly responsible for the faster ice melt at the north pole deserves investigation.

Key Words and Phrases

Human activities, fossil fuel combustion, GH gases, altitude of release, escaping terrestrial radiation, TME, jet aircraft flying.

1 Introduction

GH gases (CO_2 , water vapor, O_3 , CH_4 , CO , N_2O etc.) are contained in large quantities in emissions from fossil fuel combustion involved in a wide range of industrial and domestic human activities. The atmospheric concentrations of CO_2 , CH_4 , and tropospheric O_3 have all increased significantly as a result. By absorbing outgoing terrestrial (longwave or infrared) radiation, these gases upset the balance between the energy arriving from the sun, and energy escaping back into space; and contribute to global warming. Evidence that raising levels of GH gases in the atmosphere are warming the surface of the earth and causing other changes in the earth's climate is mounting, and has become an issue of major concern.

Different human activities cause GH gas emissions at different altitudes. Emissions from ground transportation occur at the earth's surface. Emissions from the tall chimneys of power plants occur at higher altitudes (1 km or less). Smaller private planes release at an altitude of 4, 5 km. Commercial subsonic jet aircraft release their emissions in the UT-LS (altitudes of about 13 km). Military supersonic aircraft and the small number of commercial supersonic aircraft cruise and release their emissions at higher altitudes in the stratosphere (about 20 km).

The vertical variability in the concentration of GH gases is much greater than the horizontal and temporal variability. Up to mid-tropospheric level CO_2 in the atmosphere is well mixed by fluid motions. But CO_2 released in the exhausts in UT-LS has a higher local residence time there and consequent slow mixing due to highly seasonal and latitudinal

variations in wind patterns in UT-LS. Also, water vapor and O_3 concentrations show high variability with altitude and over time.

Since the emissions are not uniformly distributed vertically, and are likely to build up locally for long residence times in some regions, it is an important problem to analyze at what altitude additional releases of GH gases will have maximum impact on global warming. In fact while discussing solar absorption (i.e., absorption of incoming solar radiation) (WMO, 1985, , page 825) states “solar absorption by a radiatively active gas (for example water vapor) will add to or ameliorate the greenhouse effect depending on the altitude of solar absorption”. We provide an analysis to show that the greenhouse effect of longwave absorption by GH gases also depends on the altitude of this absorption. We investigate both the short term and longer term effects.

For investigating the short term effects of a one time release we define an index, **TME** (transient marginal effect) which is the momentary change in the fraction of terrestrial radiation escaping the earth system when an additional unit of GH gases is released into the atmosphere. We investigate how TME varies with the altitude of release, and show that TME is highest at the altitudes of UT-LS.

Among all human activities involving fossil fuel combustion, we show that jet aircraft flying corresponds to the maximum value for TME, and that this effect from this activity lasts much longer than the effect from other human activities.

Longer term effects are investigated using a computer program for calculating radiative

forcing.

2 Model to Analyze Short Term Effects of a One Time Release

We restrict our attention to the region of the atmosphere below the LS since most of the anthropogenic emissions of GH gases occur in this region. Let E denote the fraction of terrestrial radiation escaping from the top of this region. In this section we will discuss a simple mathematical model to investigate the short term effect of an additional one time GH gases release at various altitudes, on E .

The fraction of outgoing terrestrial radiation absorbed by GH gases in the atmosphere varies substantially between different GH gas species, and the wavelength of that radiation. Each GH gas species absorbs terrestrial radiation of certain wavelengths, but remains almost transparent to other wavelengths. So, to calculate the total radiative transfer one has to integrate over the entire spectrum and add over the different GH gases. But we only want to investigate how the radiative transfer changes with the altitude of additional release. So, for the sake of our investigation, we will assume that the term *terrestrial radiation* refers to radiation of some fixed wavelength, and that *GH gas* refers to some GH gas species which is an important absorber of radiation at that wavelength. Conclusions similar to those drawn will hold for all wavelengths and GH gases, and the total radiative transfer.

Also, since we are only investigating short term effects in this section, we will ignore fluid flows and atmospheric circulations in this model. These will be taken into account when we evaluate the longer term effects. One of the aims of the simple model discussed here is to provide an intuitive justification for the conclusions to follow in subsequent sections.

We divide the region between the surface of the earth and the LS into thin *slabs* or *layers*, each slab with a homogeneous GH gas concentration. Consider a particular slab at an altitude of a from the surface of the earth. Let

$p(a) =$ probability that a beam of longwave radiation entering this slab from below crosses this slab into the next slab without being absorbed.

Since the molecular density of GH gases per unit volume in a slab decreases with increasing altitude, $p(a)$ is a monotonic increasing function of a .

Consider a long wave beam with one unit of energy entering this slab from below. The fraction $p(a)$ of it will pass to the next higher slab without being absorbed, and the remaining fraction $(1 - p(a))$ will be absorbed by GH gas molecules in our slab.

The radiatively active constituents absorb as well as emit longwave radiation. Only a part of the absorbed energy may be emitted back by this molecule, the remaining will have been truly absorbed in the sense that it remains locally in the form of an increase in the local temperature. Let

$\pi(a)$ = fraction of the infrared energy absorbed by a GH gas molecule at altitude a that remains in the form of an increase in local temperature.

The amount of absorbed energy emitted back by a GH gas molecule depends on its temperature. This is an important fact whose net effect is to reduce the longwave radiation escaping to space. As an explanation we quote from (WMO, 1985, page 823) “the fundamental cause for the reduction is the decrease in tropospheric temperature with altitude. The radiatively active gases absorb radiation emitted by the warmer (earth’s) surface (which is at 288 K on an average, and emits longwave radiation at the rate of about 390 W m^{-2}), but emit to space at the colder atmospheric temperature (which decreases with altitude to about 220 K at the top of the region we are considering, from where outgoing longwave radiation is only at the rate of 236 W m^{-2}); hence the net effect is to reduce the radiation (escaping) to space.”

That’s why $\pi(a)$ depends on the altitude a . Because of the temperature gradient with altitude, the actual value of $\pi(a)$ for a particular beam of absorbed terrestrial radiation depends on the altitude from which that beam was emitted. This altitude can be anything from 0 to a ; so in our definition, we assume that the value of $\pi(a)$ is the average of these actual values over all these altitudes from where that beam may have been emitted.

$\pi(a)$ is a monotonic increasing function of a .

The remaining $(1 - \pi(a))$ fraction of the absorbed energy is emitted by the GH gas molecule equally in all directions. So, half this emitted energy will cross this slab and enter

the next slab. The other half is directed towards the earth's surface.

Therefore of the energy in the original beam of long wave radiation entering this slab from below, a fraction of

$(1 - p(a))\pi(a)$ is absorbed by a GH gas molecule and remains locally in the form of increased local temperature

$\beta(a) = p(a) + (1 - p(a))(1 - \pi(a))/2$ escapes into the slab above

$\alpha(a) = (1 - p(a))(1 - \pi(a))/2$ is redirected towards the earth's surface

in this first iteration. The fraction of energy $\alpha(a)$ in line 3 above will eventually return along the same path in succeeding iterations.

Single Slab Model

Now consider a simple model for a single slab of air lying over the surface of the earth. For this simple model we will denote the parameters defined above by p, π, β, α . The fraction α of energy emitted by the GH gas molecule towards the earth's surface, will be emitted back by the earth's surface and enters the slab again, and will undergo the same process itself. The same process continues back and forth in repeated iterations.

Therefore the fraction of energy in the original longwave beam entering this slab from below in the first iteration that eventually escapes from the top of this slab can be approximated by

$$e = \beta(1 + \alpha + \alpha^2 + \alpha^3 + \dots) = \beta/(1 - \alpha).$$

Multislab Model

Now we will consider the general multislab model. Suppose we divide the region of the atmosphere we are considering into n slabs or layers. We make the following assumptions.

1. Each slab is a thin layer with homogeneous GH gas concentration. We assume that the slab is so thin that each beam will have at most one encounter with a GH gas molecule in it. The slabs are numbered 1 to n in increasing order of altitude above the surface of the earth.
2. For $i = 1$ to n let a_i be the altitude of slab i , $p_i = p(a_i)$, $\pi_i = \pi(a_i)$, $\alpha_i = \alpha(a_i)$, and $\beta_i = \beta(a_i)$, and $e_i = e(a_i) = \beta_i/(1 - \alpha_i)$.
3. Emissions of absorbed energy by a GH gas molecule in this slab towards the surface of the earth may in reality be absorbed by another GH gas molecule in a bottom slab and reemitted by it, or reach the surface of the earth and reemitted by it; and this process can be repeated any number of times.

Our goal is to obtain a formula for the fraction of energy in a unit energy longwave beam entering the first slab at its bottom, that eventually escapes through the top layer n

(portions of it may go back and forth in the atmosphere any number of times before this escape). This is the E we defined earlier, for the n slab model we will denote it by E_n .

Consider the two slab model, i.e., $n = 2$; and a longwave beam of unit energy entering through the bottom of the first slab. From the calculation in the single slab model we know that after passing a certain number of times back and forth between the first slab and the earth's surface, a fraction e_1 of this beam will enter the second slab.

In this quantity e_1 in the first iteration involving the second slab, $\beta_2 e_1$ escapes through the top of the second slab, and $\alpha_2 e_1$ will be directed to the earth's surface.

Of this $\alpha_2 e_1$ entering slab 1 towards the earth, $\alpha_1(\alpha_2 e_1)$ will be redirected towards slab 2, and $\beta_1(\alpha_2 e_1)$ will reach the surface of the earth and begin its upward journey again. From the single slab model we know that $e_1(\beta_1 \alpha_2 e_1)$ of this will eventually reach slab 2 from the bottom after going back and forth between slab 1 and the earth.

So, the total entering slab 2 from below in the 2nd iteration is $(\alpha_1 + \beta_1 e_1)\alpha_2 e_1$. Of this $\beta_2(\alpha_1 + \beta_1 e_1)(\alpha_2 e_1)$ escapes through the top of slab 2, and $\alpha_2(\alpha_1 + \beta_1 e_1)(\alpha_2 e_1)$ is redirected towards the earth's surface and will again enter slab 1 on its way. Repeating this argument we see that the total fraction of energy in the original beam escaping above slab 2 is

$$\beta_2 e_1 [1 + \alpha_2(\alpha_1 + \beta_1 e_1) + (\alpha_2(\alpha_1 + \beta_1 e_1))^2 + \dots]$$

$$\beta_2 e_1 / (1 - \alpha_2(\alpha_1 + \beta_1 e_1))$$

which is approximately equal to $e_2 e_1$.

Continuing this exact computation for the multislab case involving $n \geq 3$ slabs becomes very complicated. However, we can get a reasonably close approximation to E_n , the total fraction of energy escaping through the top of the n th slab by a simple argument. We notice that portions of this escaping energy may pass back and forth through each slab any number of times. Applying the single slab model to slab i , we know that the fraction of terrestrial radiation entering through the bottom of slab i that escapes through its top is e_i . Energy escaping through the top of the n th slab has to escape through the tops of every other slab. This suggests that the fraction of terrestrial radiation emitted by the earth's surface that eventually escapes through the top of the n th slab can be approximated by the product

$$E_n = e_n e_{n-1} \dots e_1.$$

Analysis

We know that $p_1 < p_2 < \dots < p_n$.

For any altitude a , $e(a) = \beta(a)/(1 - \alpha(a))$. From this it can be verified that the derivative $\frac{\partial e}{\partial p}$ is positive, therefore $e(a)$ increases as $p(a)$ increases.

We show the values of $\frac{\partial e}{\partial p}$ for some common values of p and π .

Table 1: Value of $\frac{\partial e}{\partial p}$ for

π	$p = 0.2$	$p = 0.1$
0.4	1.21	0.595
0.1	1.34	1.032

As the table indicates, it can be verified that the value of $\frac{\partial e}{\partial p}$ increases as p increases; and the rate of increase in $\frac{\partial e}{\partial p}$ with p is much higher at higher values of π .

Suppose an additional unit (by weight say) of GH gases is released into the atmosphere, say into Slab J . The immediate effect is an increase in the value of

$$d_J = \text{local molecular density of GH gases in the neighborhood of the point of release}$$

which results in a decrease in the value of p_J corresponding to slab J in the vicinity of the point of release. For all other slabs $j \neq J$ their value of p_j at that point of time is not affected. Define

Transient Marginal Effect ($\text{TME}(a_J) = \text{TME}_J$) per unit of GH gases released at this altitude a_J on E_n , = momentary change in E_n due to the release of this additional unit of GH gases into slab J .

Our aim here is to find out how TME_J varies with J .

Change in the local value of d_J is an increase; and the magnitude of this increase

increases with J , reaching its highest value at the highest altitude.

Change in the local value of p_J is a decrease; the magnitude of this decrease increases as J increases.

Change in the local value of e_J is a decrease; the magnitude of this decrease increases as J increases. The values of e_j for $j \neq J$ are not affected.

Since $E_n = e_n e_{n-1} \dots e_1$, we see that the value of E_n decreases as expected. Also

TME_J = the magnitude of momentary decrease in E_n as a result of additional releases in slab J .

The implications listed above show that TME_J increase as J increases; i.e.,

$$\text{TME}_1 < \text{TME}_2 < \dots < \text{TME}_n.$$

So, TME_J achieves its highest value when $J = n$; i.e., when the release of the additional unit of GH gases occur in the topmost slab, or in the UT-LS.

Defining $\text{TME}(a)$ as the transient marginal effect (a decrease) in E of an additional unit of GH gases released at altitude a , our analysis shows that $\text{TME}(a)$ increases with a . For altitudes a below the UT-LS there is very strong and rapid vertical mixing, so this effect is very transient at these altitudes. However, vertical mixing from UT-LS to lower altitudes is very slow, so at their altitudes this effect is likely to persist for much longer durations.

3 Analyzing Longer Term effects

The analysis in the previous section ignored atmospheric motions and zonal winds that lead to rapid mixing of air in the lower troposphere. In contrast, the UT-LS around the tropopause is a region of relative vertical stability, and emissions here will change altitude very slowly and can remain there for months or years. For this reason, among the entire range of altitudes in the region of the atmosphere we are investigating, only two different alternatives need to be compared for longer term effects, these are:

1. Longer term effects of additional GH gas releases in the UT-LS,
2. Longer term effects if the same releases occur in the lower troposphere.

Jet aircraft flying is the only human activity that releases GH gases in the UT and LS by direct injection. From (Gettelman and Baughcum, 1999) we find that a major fraction of jet aircraft flying occurs in the North Atlantic Tube (henceforth **NA Tube**) between 11 to 13 km altitude, and 30° to 90° latitude covering 25% of the earth's surface area. The total annual aviation fuel burn in the NA Tube is estimated at 100 Tg yr⁻¹ (henceforth we will refer to this as the **aircraft burn**), emissions from this burn contain 316 Tg yr⁻¹ of CO_2 and 124 Tg yr⁻¹ of H_2O (water vapor), the main GH gases to which we will restrict our attention.

In Section 2 for simplicity we considered the short term effect of a one time additional release of GH gases. The important feature of aircraft burn is that it takes place at a more

or less uniform rate throughout the year. So, for evaluating the longer term effects of aircraft burn exhaust releases, we will have to take this feature into account also.

The other region for our comparison is the lower tropospheric region between 0 to 11 km altitude (henceforth **LTR**).

The average CO_2 content in the atmosphere in the LTR is 375 ppmv, and outside the NA Tube at its altitude is 370 ppmv.

We get an estimate for the standard yearly average H_2O content in the NA Tube from Tables C3, C4, C5, C6; and in the LTR from Table C1 in (Thomas and Stamnes, 1999). The total air content in the NA Tube and LTR is also obtained from the data in these tables.

The local residence times of emissions at the altitudes of the UT-LS has been estimated to be about 1.4 years in (Warneck, 1988, page 31). In Table 1-9 of this book several references are quoted estimating this residence time to be between 0.8 years to 2.2 years using a variety of techniques), and over a year in (Holton, et al. 1995) We will assume that this local residence time is one year on an average.

Since jet aircraft flying occurs at an almost constant rate throughout the year, its contribution to the GH gas content in the NA Tube is equivalent to a permanent addition of a quantity contained in one year's aviation fuel burn in the tube, i.e., 316 Tg of CO_2 (43.3×10^{35} CO_2 molecules) and 124 Tg of H_2O (41.5×10^{35} H_2O molecules).

CO_2 residence time in LTR is over 100 years, so we treat it as essentially infinity. The average H_2O residence time in LTR is 10 days (Warneck, 1988). So if the aircraft burn were

to take place in the LTR instead of the NT Tube, it contributes 316 Tg of additional CO_2 (43.3×10^{35} molecules) to the LTR in one year's time. Since the average residence time of H_2O in the LTR is only 10 days, this burn will contribute a permanent addition of $(124/36.5)$ Tg of H_2O (1.15×10^{35} molecules) to the LTR.

The following table gives some data and summarizes this information.

Table 2: Perturbations caused by aircraft burn

Data	For region	
	NA Tube	LTR
1. Volume (cm ³)	2.55×10^{23}	5.61×10^{24}
2. Total molecules in air	16.234×10^{41}	0.86×10^{44}
3. No. molecules not counting contribution of aircraft burn, of		
<i>CO</i> ₂	6×10^{38}	32.25×10^{39}
<i>H</i> ₂ <i>O</i>	29.94×10^{36}	27×10^{40}
4. No. molecules counting contribution of aircraft burn*, if burned in NA Tube, of		
<i>CO</i> ₂	6×10^{38} $+43.3 \times 10^{35}$	32.25×10^{39}
<i>H</i> ₂ <i>O</i>	29.94×10^{36} $+41.5 \times 10^{35}$	27×10^{40}
5. No. molecules counting contribution of aircraft burn*, if burned in LTR, of		
<i>CO</i> ₂	6×10^{38}	32.25×10^{39} $+43.3 \times 10^{35}$

The radiative forcing under the two perturbations in lines 4, 5 of this table have been computed with Seiji Kato's computer program. This is a limited evaluation that took only the H_2O perturbations into account, but did not have the effects due to the formation of any condensation trails and cirrus clouds. It estimated the total amount of terrestrial radiation escaping above 13 km altitude to be

300.60 Under the perturbation introduced by the aircraft burn
in the NA Tube

300.95 Under the perturbation introduced if the aircraft burn
were to take place in the LTR.

This shows that the present aircraft burn in the NA Tube has a slightly higher global warming potential than if the same burn were to occur at lower altitudes. It indicates that the northern mid-latitudes where majority of jet aircraft releases occur today is a sensitive region for such releases.

4 Related Topics for Further Study

Air travel has become an integral part of modern life and has been increasing at 5 to 9% per year (IPCC, 1999). It has become the principal mode of transportation for trips longer than a few hundred miles, and made our earth a small planet. But its speed and convenience comes at a price as indicated by the results above. A commercial passenger jet airline burns

about 8 times the fuel that a ground vehicle burns per revenue passenger km. In addition to releasing GH gases in a sensitive region, it also releases NO_x, SO_x which are chemically active species that alter the chemistry of this region.

A related topic is the melting of ice at both the north and south poles. But polar ice melt is faster at the north pole than at the south pole. In fact vessels traveling along the northern passage have noticed on several occasions that several km^2 area near the north pole has become liquid water ocean in recent years, whereas the area around the south pole has remained frozen so far. An important problem for further investigation is to determine whether the large volume of jet aircraft releases in northern mid-latitudes are at least partly responsible for the faster ice melt at the north pole.

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